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S-brane thermodynamics

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Abstract: The description of string-theoretic s-branes at $g_s = 0$ as exact worldsheet CFTs with a $\lambda \cosh X^0$ or $\lambda e^{\pm X^0}$ boundary interaction is considered. Due to the imaginary-time periodicity of the interaction under $X^0 \to X^0 + 2\pi i$, these configurations have intriguing similarities to black hole or de Sitter geometries. For example, the open string pair production as seen by an Unruh detector is thermal at temperature $T = 1/4\pi$. It is shown that, despite the rapid time dependence of the s-brane, there exists an exactly thermal mixed state of open strings. The corresponding boundary state is constructed for both the bosonic and superstring cases. This state defines a long-distance euclidean effective field theory whose light modes are confined to the s-brane. At the critical value of the coupling $\lambda = 1/2$, the boundary interaction simply generates an SU(2) rotation by π from Neumman to Dirichlet boundary conditions. The $\lambda = 1/2$ s-brane reduces to an array of sD-branes (D-branes with a transverse time dimension) on the imaginary time axis. The long range force between a (bosonic) sD-brane and an ordinary D-brane is shown from the annulus diagram to be 11/12 times the force between two D-branes. The linearized time-dependent RR field $F_{p+2} = dC_{p+1}$ produced by an sDp-brane in superstring theory is explicitly computed and found to carry a half unit of s-charge $Q_s = \int_{\Sigma_{8-p}} *F_{p+2} = 1/2$, where Σ_{8-p} is any transverse spacelike slice.

Keywords: p-branes, Tachyon Condensation.

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1. Introduction

A spacelike brane, or s-brane, is much like an ordinary brane except that one of its transverse dimensions includes time. S-branes arise as time-dependent, soliton-like configurations in a variety of field theories. In string theory, the potential for the open string tachyon field leads to s-branes [1] in a time-dependent version of the construction [2] of D-branes as solitons of the open string tachyon. These s-branes can be thought of as the creation and

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subsequent decay of an unstable brane. They are of interest as relatively simple examples of time-dependent string backgrounds. Some of the recent investigations can be found in [3]–[18] and related earlier work is in [19, 20].

An elegant worldsheet construction of a family of s-branes was given in the classical $g_s = 0$ limit by Sen [3]. This construction employs an analytic continuation of the conformally-invariant boundary Sine-Gordon model. It describes the tachyon field on an unstable bosonic D-brane by the boundary interaction on the string worldsheet

$$S_{\text{boundary}} = \lambda \int d\tau \cosh \frac{X^0(\tau)}{\sqrt{\alpha'}} \,, \tag{1.1}$$

where X^0 is the time coordinate. Qualitatively similar constructions were also given for the superstring [4]. In this paper we will describe some surprising and intriguing properties of these s-branes.

A salient feature of time-dependent backgrounds is that there is in general no preferred vacuum and particle production is unavoidable. We study the open string vacua on an s-brane and find that they have somewhat mysterious thermal properties reminiscent of black hole or de Sitter vacua. For the case (1.1) there is open string pair production with a strength characterized by the Hagedorn temperature [12, 17]

$$T_H = \frac{1}{4\pi\sqrt{\alpha'}} \,. \tag{1.2}$$

Mathematically, the temperature arises from the periodicity of the boundary interaction (1.1) in imaginary time. Physically, we show that (for the quantum state of a brane created with no incoming string excitations) an Unruh detector will see a thermal bath. We further show that at late times the exact Green function approaches the thermal Green function plus asymptotically vanishing corrections.

The appearance of the Hagedorn temperature signals a breakdown of string perturbation theory [12, 14, 21, 22]. Describing the mechanism which cuts off this divergence is an interesting problem which we will not address in this paper.[1] Herein we simply avoid the problem by working at $g_s = 0$. Interestingly, we find that the Hagedorn problem disappears in the $\lambda = 1/2$ sD-brane case discussed below.[2]

In addition to the pure vacuum states mentioned above, we construct a series of mixed thermal states with temperatures

$$T = \frac{1}{2\pi n\sqrt{\alpha'}} \tag{1.3}$$

for positive integer n. Ordinarily it makes no sense to discuss a thermal state in a highly time-dependent background. However, because of the euclidean time periodicity of the

[1]The divergence might be controlled by lowering the s-brane temperature either with the addition of an electric field [23, 24] or a null linear dilaton [25].

[2]This can be seen both from the absence of on-shell open strings and the vanishing of the projection of the boundary state onto the massless winding state in the closed string channel associated to the Hagedorn divergence.

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interaction (1.1), we can construct mixed states whose Green functions have exact thermal periodicity at all times. Physically, it is natural to expect open strings on an s-brane to be in a mixed state, since they are correlated with the closed string modes whose energy was needed to create the s-brane in the first place.

Ordinary branes are usefully characterized at long distances by a long-distance effective field theory. It is rather subtle to define such an effective theory for an s-brane because there is no time-translation invariant ground state around which to define and expand the low-lying excitations. One may try to define a long-distance effective field theory as the euclidean theory which reproduces the long-distance equal time correlators on the s-brane. In general these correlators depend strongly on the quantum state of the fields on the s-brane, and will not behave like those of any euclidean field theory whose dimension is that of the s-brane. However, we show that the thermal state of the s-brane gives an effective field theory which is essentially a (twisted) compactification of the unstable brane whose creation/decay comprises the s-brane. Ultimately, this euclidean effective field theory may play an interesting role in timelike holographic duality.

The thermal s-brane states can be succinctly characterized by a CFT boundary state. The thermal boundary state at temperature $T = 1/2\pi n\sqrt{\alpha'}$ differs from the zero-temperature boundary state of [3] by a periodic identification $\phi \sim \phi + 2\pi n$ of the euclidean timelike scalar $\phi = -iX^0$. The thermal s-brane boundary state contains closed strings with winding in the ϕ direction. These thermal boundary states enable efficient evaluation of various worldsheet string diagrams.

At the special value of the coupling $\lambda = 1/2$ appearing in (1.1), a dramatic simplification occurs. For any λ the boundary interaction (1.1) generates a right-moving rotation by $2\pi\lambda$ in the SU(2) level one current algebra generated by ∂X^0, $\cosh X^0$ and $i \sinh X^0$. When $\lambda = 1/2$ the rotation is by π, which simply transforms the Neumann boundary state into a Dirichlet boundary state. The s-brane then degenerates into a periodic array of sD-branes (i.e. D-branes with a Dirichlet boundary condition on the time coordinate) located on the imaginary time axis at $\phi = m\pi$ for odd integer m. This gives the precise relation between the Dirichlet type boundary states discussed in [1] and the Sine-Gordon type boundary states discussed in [3].[3] In this $\lambda = 1/2$ limit there are no on-shell open string states,[4] but the on-shell closed string states determined from the boundary state remain. This remaining closed string configuration has the unusual property that its total energy is of order $1/g_s$ and its behavior can be determined from open string calculations on the sD-brane. The annulus diagram connecting one (bosonic) D-brane and one s-brane is computed for arbitrary λ using the boundary state. At $\lambda = 1/2$, the long-distance force between a D-brane and an sD-brane is shown to be 11/12 times the force between two D-branes (which corresponds to $\lambda = 0$).[5]

[3]Reference [1] only considered sD-branes on the real time axis. As discussed therein, these have problems both with the dominant energy condition and the existence of a well-defined string perturbation expansion. Both of these problems seem to be resolved by moving the sD-brane to imaginary time. In [14] it was shown that, for the bosonic string, the theory with $\lambda = -1/2$ has an sD-brane at real time.

[4]Off-shell open string states still appear in the euclidean effective field theory.

[5]This might appear to contradict the claim [3] that $\lambda = 1/2$ is the trivial closed string vacuum. The

This paper is organized as follows. Section 2 describes various s-brane vacua and their properties in a minisuperspace approximation which treats the open strings as quantum fields with a time-dependent $\cosh t$ or $e^{\pm t}$ mass. Most of the important behavior follows from the euclidean periodicity of the interaction, which is an exact property of the world-sheet CFT. In section 2.1 we review the "half s-brane" corresponding to brane decay, which is described by a (in some respects simpler) boundary interaction $\int e^{X^0} d\tau$ instead of (1.1). We recall that open string pair creation is characterized by the Hagedorn temperature $T_H = 1/4\pi\sqrt{\alpha'}$ [12]. In section 2.2 we consider the time-reverse process of brane creation, for which a natural state is one with no incoming open strings. In section 2.3 we move on to the full s-brane (1.1), whose linearized solutions are Mathieu functions. We use these solutions in section 2.4 to describe two vacua of the full s-brane with no particles in the far past and the far future, respectively. In section 2.5 a time-reversal invariant s-brane vacuum is defined by the condition that there is no particle flux in the middle of the s-brane at $t = 0$. The resulting positive frequency modes are found to be bounded in the lower half plane $t \to -i\infty$. This vacuum can therefore be obtained by analytic continuation from euclidean space. Other possible vacua are described in section 2.6.

In section 3 we discuss the thermal properties of s-branes. In section 3.1 it is shown that thermal Green functions at temperature $T = 1/2\pi n\sqrt{\alpha'}$ can be obtained by analytic continuation from a periodically identified euclidean section. The minkowskian mixed thermal state (or density matrix) which reproduces these Green functions is explicitly constructed. Thermal properties of certain pure vacuum states arising from the imaginary-time periodicity of (1.1) are also demonstrated. For example, in section 3.2 it is shown that an Unruh detector in the vacuum with no incoming particles measures a temperature T_H during brane creation. Furthermore, in section 3.3 it is found that at late times the correlators in the pure state approach thermal correlators plus asymptotically vanishing corrections. This suggests that branes are naturally produced in something close to a thermal state.

Section 4 defines the notion of long-distance effective field theory for an s-brane. This effective theory is determined as the euclideanization and time compactification, with twisted fermion boundary conditions and a periodic boundary tachyon interaction on the worldsheet, of the theory on the unstable brane whose creation/decay describes the s-brane. Long-distance modes are related to zero modes of this compactified theory.

In section 5.1 the boundary states which generate the finite-temperature string correlators on an s-brane are constructed. They differ from the usual expression by a periodic identification of euclidean time, which allows for winding modes in the closed string channel. The superstring is discussed in section 5.2, and the allowed temperatures are shown to be

$$T = \frac{1}{2\pi n\sqrt{2\alpha'}} \,. \tag{1.4}$$

interesting resolution of this apparent conflict, discussed in section 6.1 in some detail, is that when there is time-dependence, the boundary state does not uniquely determine the closed string fields. The implicit prescription adopted in [3] differs from the one used herein, which latter amounts to the use of the Feynman propagators obtained by analytic continuation from euclidean space.

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In section 6 we turn to the special case of $\lambda = 1/2$, in which the boundary interaction becomes an SU(2) rotation by π. In this case the s-brane collapses to an array of sD-branes on the imaginary time axis at $t = m\pi i$ for odd integer m. In section 6.1 we explain an ambiguity in the propagator used to obtain the spacetime closed string fields from the boundary state, which is due to the appearance of on-shell states in the boundary state. It is shown that the analytic continuation of the Feynman propagator gives a non-zero answer even when the support of the boundary state moves off the real time axis. The summation over the sD-branes m is performed in a simple example to give a closed-form expression for a massless closed string field. In section 6.2 we compute the RR field emanating from an sD-brane and integrate it over a transverse spacelike surface to find the s-charge. Long range dilaton/graviton fields are computed in section 6.3. In section 6.4 we compute the annulus diagram connecting an ordinary D-brane to an s-brane for general λ, using old results on the boundary Sine-Gordon model [26] . We find that the long distance force between a D-brane and an s-brane has a coefficient of $23 + \cos(2\pi\lambda)$, indicating that the force between an sDp-brane ($\lambda = 1/2$) and a D-brane is 11/12 times the force between two ordinary D-branes. In 6.5 we extend the annulus computation to finite temperature. In 6.6 we discuss the relation between sD-branes and D-instantons. Finally, we conclude in section 7 with speculations on timelike holography.

Results overlapping with those of this paper will appear in [21].

2. Quantum vacuum states

We wish to understand the dynamics of the open string worldsheet theory with a time-dependent tachyon

$$S = -\frac{1}{4\pi}\int_{\Sigma_2} d^2\sigma \partial^a X^\mu \partial_a X_\mu + \int_{\partial\Sigma_2} d\tau\ m^2(X^0)\,, \tag{2.1}$$

where here and henceforth we set $\alpha' = 1$. For the open bosonic string $m^2 = T$ where T is the spacetime tachyon, while for the open superstring $m^2 \sim T^2$ after integrating out worldsheet fermions. We use the symbol m^2 to denote the interaction because the coupling (among other effects) imparts a mass to the open string states. We consider three interesting cases described by the marginal interactions [3, 12, 17, 27][6]

$$m_+^2(X^0) = \frac{\lambda}{2}e^{X^0} \tag{2.2}$$

$$m_-^2(X^0) = \frac{\lambda}{2}e^{-X^0} \tag{2.3}$$

$$m_s^2(X^0) = \lambda\cosh X^0\,. \tag{2.4}$$

[6]For the bosonic string one could also consider a $\sinh X^0$ interaction, although we do not do so here. In the bosonic theory $\cosh X^0$ describes a process in which a tachyon rolls up the barrier and then back down the same side, while $\sinh X^0$ describes the tachyon rolling over the barrier. The bosonic $\sinh X^0$ interaction is challenging because it ventures into the unbounded side of the tachyon potential. The superstring potential is always positive, so there is no analog of a $\sinh X^0$ type interaction.

The first case m_+^2 describes the process of brane decay, in which an unstable brane decays via tachyon condensation. The second case describes the time-reverse process of brane creation, in which an unstable brane emerges from the vacuum. The final case describes an s-brane, which is the process of brane creation followed by brane decay. Brane decay (2.2) (creation (2.3)) can be thought of as the future (past) half of an s-brane, i.e. as the limiting case where the middle of the s-brane is pushed into the infinite past (future).

An exact CFT analysis of these s-brane theories should be possible [3, 12, 17, 27], and some exact results are given in sections 5 and 6. However in this and the next section we shall confine ourselves to the minisuperspace analysis [12] in which the effect of the interaction is simply to give a time-dependent shift (given by (2.2)–(2.4)) to the masses of all the open string states. The range of validity of the minisuperspace approximation is unclear, although some evidence in favor of its validity at high frequencies was found in [17]. However, most of the results of the next two sections follow from the periodicity in imaginary time, which is an exact property of the CFTs defined by (2.2)–(2.4) , so we expect our conclusions to be qualitatively correct.

In the minisuperspace approximation only the zero-mode dependence of the interaction $m^2(X^0)$ is considered. In this case we can plug in the usual mode solution for the free open string with oscillator number N to get an effective action for the zero modes

$$S = \int d\tau \left[-\frac{1}{4}\dot{x}^\mu \dot{x}_\mu + (N-1) + 2m^2(x^0) \right] . \tag{2.5}$$

This is the action of a point particle with a time dependent mass. Here $x^\mu(\tau)$ is the zero mode part of $X^\mu(\sigma,\tau)$, and the second term in (2.5) is an effective contribution from the oscillators, including the usual normal ordering constant. From (2.5) we can write down the Klein-Gordon equation for the open string wave function $\phi(t,\vec{x})$,

$$\left(\partial^\mu\partial_\mu - 2m^2(t) - (N-1)\right)\phi(t,\vec{x}) = 0\,, \tag{2.6}$$

where $(t,\vec{x})$ are the spacetime coordinates corresponding to the worldsheet fields $(X^0,\vec{X})$. This is the equation of motion for a scalar field with time-dependent mass.

At this point, we should make a few remarks about field theories with time dependent mass. Time translation invariance has been broken, so energy is not conserved and there is no preferred set of positive frequency modes. This is a familiar circumstance in the study of quantum field theories in time-dependent backgrounds which leads to particle creation. The probability current $j_\mu = i(\phi^*\partial_\mu\phi - \partial_\mu\phi^*\phi)$ is conserved, allowing us to define the Klein-Gordon inner product

$$\langle f|g\rangle = i\int_\Sigma d\Sigma^\mu (f^*\partial_\mu g - \partial_\mu f^* g) \tag{2.7}$$

where Σ is a spacelike slice. This norm does not depend on the choice of Σ if f and g solve the wave equation. Normalized positive frequency modes are chosen to have $\langle f|f\rangle = 1$. Negative frequency modes are complex conjugates of positive frequency modes, with $\langle f^*|f^*\rangle = -1$. There is a set raising and lowering operators associated to each choice of mode decomposition — these operators obey the usual oscillator algebra if the corresponding modes are normalized with respect to (2.7). We also define a vacuum state associated to each mode decomposition — it is the state annihilated by the corresponding lowering operators.

2.1 The $|in\rangle_+$ vacuum for brane decay

In this section we review some results of [12] on a scalar field with mass

$$m_+^2(t) = \frac{\lambda}{2} e^t \,, \tag{2.8}$$

describing open strings on a decaying brane. A natural vacuum in this case is that with no particles present in the far past: we shall denote this state $|in\rangle_+$.

Expanding ϕ in plane waves

$$\phi(t, \vec{x}) = e^{i\vec{p}\cdot\vec{x}} u(t) \tag{2.9}$$

the wave equation becomes

$$(\partial_t^2 + \lambda e^t + \omega^2) u = 0 \,, \qquad \omega^2 = p^2 + N - 1 \,. \tag{2.10}$$

This is a form of Bessel's equation. It has normalized, positive frequency solutions[7]

$$u_+^{\rm in} = \lambda^{i\omega} \frac{\Gamma(1-2i\omega)}{\sqrt{2\omega}} J_{-2i\omega}(2\sqrt{\lambda} e^{t/2}) \,. \tag{2.11}$$

These solutions have been chosen because they approach flat space positive frequency plane waves in the far past $t \to -\infty$,

$$u_+^{\rm in} \sim \frac{1}{\sqrt{2\omega}} e^{-i\omega t} \,. \tag{2.12}$$

We will also consider the wave functions

$$u_+^{\rm out} = \sqrt{\frac{\pi}{2}} (ie^{2\pi\omega})^{-1/2} H^{(2)}_{-2i\omega}(2\sqrt{\lambda} e^{t/2}) \tag{2.13}$$

that are purely positive frequency in the far future $t \to +\infty$,

$$u_+^{\rm out} \sim \frac{\lambda^{-1/4}}{\sqrt{2}} \exp\left\{ -t/4 - 2i\sqrt{\lambda} e^{t/2} \right\} . \tag{2.14}$$

These are related to the previous set of wave functions by a Bogolubov transformation

$$u_+^{\rm out} = a u_+^{\rm in} + b u_+^{{\rm in}*} \tag{2.15}$$

---

[7] A + (−) subscript on a wave function denotes solutions with $m^2 = m_+^2$ ($m^2 = m_-^2$) during brane decay (creation). A wave function without a subscript refers to solutions for full s-brane $m^2 = m_s^2$. A superscript *in*, *out* and 0 on a wave function denotes solutions that are purely positive frequency when $t \to -\infty$, $t \to +\infty$ or $t = 0$. The wave functions u depend on $\vec{p}$ as well, although we will typically suppress momentum indices.

whose coefficients

$$a = e^{2\pi\omega+\pi i/2}b^* = \sqrt{\omega\pi}e^{\pi\omega-\pi i/4}\left(\frac{\lambda^{-i\omega}}{\sinh 2\pi\omega\Gamma(1-2i\omega)}\right) \tag{2.16}$$

obey the usual unitarity relation $|a|^2 - |b|^2 = 1$. All solutions of the wave equation during brane decay vanish exponentially in the far future (but not in the far past) because the mass is growing exponentially.

The relation (2.15) between in and out modes implies as usual the relation between in and out creation and annihilation operators

$$a^{\rm in} = aa^{\rm out} + b^*(a^{\rm out})^\dagger\,. \tag{2.17}$$

From this, the condition that $a^{\rm in}|in\rangle = 0$ implies that $|in\rangle$ is a squeezed state

$$|in\rangle_+ = \Pi_{\vec{p}}(1-|\gamma|^2)^{1/4}\exp\left\{-\frac{1}{2}\gamma(a_{\vec{p}}^{\rm out\dagger})^2\right\}|out\rangle_+\,, \qquad \gamma = b^*/a\,. \tag{2.18}$$

Physically, this is the statement that particles are produced during brane decay: if we start in a state with no particles at $t \to -\infty$, there will be many particles at time $t \to +\infty$. We should emphasize here that γ is a function of $\vec{p}$

$$\gamma = \frac{b^*}{a} = -ie^{-2\pi\omega} \tag{2.19}$$

that decreases exponentially as the energy ω increases. In particular, this implies that the $|in\rangle_+$ and $|out\rangle_+$ vacua become identical at very short distances. The density of particles with momentum $\vec{p}$ is

$$n_{\vec{p}} = |\gamma|^2 = e^{-4\pi\omega}\,. \tag{2.20}$$

Despite the fact that (2.18) is a pure state, this is precisely the Boltzmann density of states at temperature $T_H = 1/4\pi$. In string units, T_H is the Hagedorn temperature. The fact that this "temperature" is so high means that g_s corrections are likely qualitatively important even for $g_s \to 0$ [12] , but we do not consider these here.

The appearance of the temperature T_H here is ultimately due to the euclidean periodicity of the interactions (2.2)–(2.4). This will lead to other thermal properties as described in the next section. Since the thermal periodicity is an exact property of the worldsheet CFT (2.1), we expect this behavior to persist beyond the minisuperspace approximation considered here.

2.2 The $|in\rangle_-$ vacuum for brane creation

Solutions u_- of the wave equation during brane creation are related to those (u_+) during brane decay by time reversal. In particular,

$$u_-^{\rm out}(t) = u_+^{\rm in}(-t)^* \tag{2.21}$$

becomes a plane wave in the far future $t \to +\infty$, and

$$u_-^{\rm in}(t) = u_+^{\rm out}(-t)^* = \sqrt{\frac{\pi}{2}}(ie^{2\pi\omega})^{1/2}H^{(1)}_{-2i\omega}(2\sqrt{\lambda}e^{-t/2}) \tag{2.22}$$

becomes purely positive frequency in the far past $t \to -\infty$

$$u_-^{\text{in}} \sim \frac{\lambda^{-1/4}}{\sqrt{2}} \exp\left\{t/4 + 2i\sqrt{\lambda}e^{-t/2}\right\}. \tag{2.23}$$

These two solutions are related by the Bogolubov transformation

$$u_-^{\text{in}} = a^* u_-^{\text{out}} + b^* u_-^{\text{out}*}, \tag{2.24}$$

where a and b are given by (2.16). Wave functions during brane creation vanish exponentially in the far past but not in the far future, because the masses are infinite in the far past.

The natural vacuum state during brane creation is not, however, the time reverse $\mathcal{T}$ of the in state for brane decay $\mathcal{T}|in\rangle_+$. The latter state has particles present in the far past with infinite masses. Indeed it would cost an infinite amount of energy to prepare such an initial state. Rather, the natural in state $|in\rangle_-$ for brane creation has no particles in the far past and is the time reverse $\mathcal{T}|out\rangle_+$ of the out vacuum for brane decay. We can write $|in\rangle_-$ in terms of the free out operators as

$$|in\rangle_- = \Pi_{\vec{p}}(1-|\gamma|^2)^{1/4} \exp\left\{\frac{1}{2}\gamma^*(a_{\vec{p}}^{\text{out}\,\dagger})^2\right\}|out\rangle_- \tag{2.25}$$

where γ is given by (2.19). The spectrum in the region $t \to +\infty$ is just the free spectrum of the unstable D-brane, and (2.25) is a pure state of open string excitations. Despite this fact we shall see in section 3 that (2.25) closely resembles a thermal state at temperature T_H. Indeed we shall see that the results of measurements done after brane creation differ from thermal results by asymptotically vanishing amounts.

2.3 Full s-brane modes

For the full s-brane potential (2.4), the Klein-Gordon equation is

$$(\partial_t^2 + 2\lambda\cosh t + \omega^2)u = 0. \tag{2.26}$$

This is a form of Mathieu's equation. We will now summarize a few useful properties of the solutions — see e.g. [28, 29, 30] for more detail. The solutions are generalized Mathieu functions, which can be written as[8]

$$u = Ae^{-i\tilde{\omega}t}P(t) + Be^{i\tilde{\omega}t}P(-t) \tag{2.27}$$

where the function

$$P(t) = \sum_{r=-\infty}^{\infty} c_{2r}e^{rt} \tag{2.28}$$

is periodic in imaginary time $P(t) = P(t+2\pi i)$. The constants $\tilde{\omega}$ and c_{2r} obey complicated recursive formulae and are typically computed numerically. Although it is not obvious from (2.27), all solutions u vanish exponentially in the far past and the far future. This is because open string modes get very massive far from the interior of the brane.

[8]This form for u differs from the standard convention for Mathieu functions. Our $\tilde{\omega}$ is related to the standard characteristic exponent (often denoted ν in the literature) by $\nu = -2i\tilde{\omega}$.

Solutions to Mathieu's equation are typically classified by their behavior with respect to imaginary time $\tau = it$. A solution is bounded on the τ axis only if $\tilde{\omega}$ is purely imaginary. In this case the solution is called *stable* - otherwise it is *unstable*. Solutions are stable only for certain regions of the (ω^2, λ) plane. For large positive ω^2 — the case of interest — $\tilde{\omega}$ is real and all solutions are unbounded on the real τ axis. However, there is a unique solution that vanishes as $\tau \to +\infty$ – this is the solution with $B = 0$. We will see later that this solution is naturally associated to the state with no particles in the interior of the s-brane at $t = 0$.

It is useful to assemble Mathieu functions in the form [30] analogous to Bessel functions

$$
\begin{aligned}
J\left(-2i\tilde{\omega}, \frac{t}{2}\right) &= \equiv e^{-i\tilde{\omega}t} P(t) = \sum_{n=-\infty}^{\infty} \phi\left(n - i\tilde{\omega}\right) e^{(n-i\tilde{\omega})t} \,, \\
H^{(1)}\left(-2i\tilde{\omega}, \frac{t}{2}\right) &\equiv \frac{J(2i\tilde{\omega}, t/2) - e^{-2\pi\tilde{\omega}} J(-2i\tilde{\omega}, t/2)}{\sinh 2\pi\tilde{\omega}} \,, \\
H^{(2)}\left(-2i\tilde{\omega}, \frac{t}{2}\right) &\equiv \frac{J(2i\tilde{\omega}, t/2) - e^{2\pi\tilde{\omega}} J(-2i\tilde{\omega}, t/2)}{-\sinh 2\pi\tilde{\omega}} \,.
\end{aligned} \tag{2.29}
$$

They have asymptotic behavior

$$
\left.\begin{aligned}
H^{(1)}\left(-2i\tilde{\omega}, \frac{t}{2}\right) &\to \frac{\lambda^{-1/4}}{\sqrt{\pi}} e^{-\pi\tilde{\omega}} \exp\left(-\tfrac{t}{4} + 2i\sqrt{\lambda} e^{t/2} - i\tfrac{\pi}{4}\right) \\
H^{(2)}\left(-2i\tilde{\omega}, \frac{t}{2}\right) &\to \frac{\lambda^{-1/4}}{\sqrt{\pi}} e^{\pi\tilde{\omega}} \exp\left(-\tfrac{t}{4} - 2i\sqrt{\lambda} e^{t/2} + i\tfrac{\pi}{4}\right)
\end{aligned}\right\} \quad \text{as } t \to +\infty \,. \tag{2.30}
$$

Mathieu's equation is invariant under $t \to -t$, so $J(-2i\tilde{\omega}, -t/2)$ is also a solution. Under $t \to t + 2\pi i$, $J(-2i\tilde{\omega}, -t/2)$ picks up the phase $e^{-2\pi\tilde{\omega}}$, so it must be proportional to $J(2i\tilde{\omega}, t/2)$

$$
J\left(2i\tilde{\omega}, \frac{t}{2}\right) = \chi J\left(-2i\tilde{\omega}, -\frac{t}{2}\right) \tag{2.31}
$$

where the proportionality factor χ is related to ϕ by

$$
\chi = \frac{\phi(n + i\tilde{\omega})}{\phi(-n - i\tilde{\omega})} = \frac{\phi(i\tilde{\omega})}{\phi(-i\tilde{\omega})}. \tag{2.32}
$$

The coefficients $\phi(\tau)$ can be computed using the formula

$$
\begin{aligned}
\phi(\tau) &= \frac{1}{\Gamma(1 + \tau + i\omega)\Gamma(1 + \tau - i\omega)} \sum_{n=0}^{\infty} (-1)^n \lambda^{2n+\tau} A_\tau^{(n)} \,, \\
A_\tau^{(0)} &= 1 \,, \\
A_\tau^{(\lambda)} &= \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \cdots \sum_{p_\lambda=0}^{\infty} a_{\tau+p_1} a_{\tau+p_1+p_2} \cdots a_{\tau+p_1+\cdots+p_\lambda} \,, \\
a_\tau &= \frac{1}{(1 + \tau + i\omega)(1 + \tau - i\omega)(2 + \tau + i\omega)(2 + \tau - i\omega)} \,.
\end{aligned} \tag{2.33}
$$

One can analyze the behavior of $H^{(i)}(-2i\tilde{\omega}, t/2)$ as $t \to -\infty$ using the relation

$$H^{(1)}\left(-2i\tilde{\omega}, \frac{t}{2}\right) = \frac{1}{2\sinh 2\pi\tilde{\omega}} \times \tag{2.34}$$
$$\times \left[\left(\chi - \frac{1}{\chi}\right) H^{(1)}\left(-2i\tilde{\omega}, -\frac{t}{2}\right) + \left(\chi - \frac{e^{-4\pi\tilde{\omega}}}{\chi}\right) H^{(2)}\left(-2i\tilde{\omega}, -\frac{t}{2}\right)\right].$$

When λ is small, we can use the expansion

$$\omega^2 = \tilde{\omega}^2 + \frac{2\lambda^2}{4\tilde{\omega}^2 + 1} + \frac{(20\tilde{\omega}^2 - 7)\lambda^4}{2(4\tilde{\omega}^2 + 1)^3(\tilde{\omega}^2 + 1)} + \cdots$$

to compute $\tilde{\omega}$.

2.4 $|in\rangle_s$ and $|out\rangle_s$ s-brane vacua

For our full s-brane, the incoming and outgoing positive frequency wave functions are normalized as

$$u^{\rm in}(t) = \sqrt{\frac{\pi}{2}}(ie^{2\pi\tilde{\omega}})^{1/2} H^{(1)}\left(-2i\tilde{\omega}, -\frac{t}{2}\right), \tag{2.35}$$

$$u^{\rm out}(t) = \sqrt{\frac{\pi}{2}}(ie^{2\pi\tilde{\omega}})^{-1/2} H^{(2)}\left(-2i\tilde{\omega}, \frac{t}{2}\right). \tag{2.36}$$

The in (out) vacuum, which has no incoming (outgoing) particles, is defined by the condition

$$a^{\rm in}|in\rangle_s = 0 = a^{\rm out}|out\rangle_s\,. \tag{2.37}$$

The relation between in and out modes is determined from (2.34) to be

$$\begin{aligned} u^{\rm in}(t) &= \frac{1}{2\sinh 2\pi\tilde{\omega}}\left[i\left(e^{2\pi\tilde{\omega}}\chi - \frac{e^{-2\pi\tilde{\omega}}}{\chi}\right) u^{\rm out}(t) + \left(\chi - \frac{1}{\chi}\right) u^{out*}(t)\right] \\ &= \alpha u^{\rm out}(t) + \beta u^{out*}(t)\,, \end{aligned} \tag{2.38}$$

where α, β are the Bogolubov coefficients

$$\alpha = \frac{i}{2\sinh 2\pi\tilde{\omega}}\left(e^{2\pi\tilde{\omega}}\chi - \frac{e^{-2\pi\tilde{\omega}}}{\chi}\right), \qquad \beta = \frac{1}{2\sinh 2\pi\tilde{\omega}}\left(\chi - \frac{1}{\chi}\right). \tag{2.39}$$

Although the dependence of $\tilde{\omega}, \chi$ on ω, λ is in general quite complicated, it is known that either $e^{2\pi\tilde{\omega}}$ is real and χ is of unit modulus, or χ is real and $e^{2\pi\tilde{\omega}}$ is of unit modulus. It follows that the Bogolubov coefficients satisfy the unitarity condition $|\alpha|^2 - |\beta|^2 = 1$. In the case of interest, ω is large and $\lambda \ll \omega$, we have

$$\tilde{\omega} = \omega\left[1 + O\left(\frac{\lambda^2}{\omega^4}\right)\right]. \tag{2.40}$$

Using (2.32) and (2.33), we obtain the expansion for χ

$$\chi = \frac{\Gamma(1 - 2i\omega)}{\Gamma(1 + 2i\omega)}\lambda^{2i\omega}\left[1 + O\left(\frac{\lambda^2}{\omega^2}\right)\right]. \tag{2.41}$$

To leading order in $O(\lambda^2/\omega^2)$, the Bogolubov coefficients are

$$\alpha = \frac{\sin(\theta + 2\pi i\omega)}{\sinh 2\pi\omega}\,, \qquad \beta = -i\frac{\sin\theta}{\sinh 2\pi\omega}\,, \qquad e^{i\theta} = \lambda^{-2i\omega}\frac{\Gamma(1+2i\omega)}{\Gamma(1-2i\omega)}\,. \tag{2.42}$$

Sub-leading terms can be computed order by order in $1/\omega^2$ using the methods of [30], although we shall not need them here.

2.5 The $|0\rangle_s$ euclidean s-brane vacuum

If we take $\lambda \to 0$ there is a long region around $t = 0$, of duration $\ln\lambda$, in which the interaction can be neglected and we just have an ordinary unstable brane. There is then a natural $|0\rangle_s$ vacuum in which there are no particles present at $t = 0$. This will later be identified with a vacuum obtained by analytic continuation from euclidean space. It is associated with the wave functions

$$u^0(t) = \sqrt{\frac{4\pi\chi}{\sinh 2\pi\tilde{\omega}}}\, J\left(-2i\tilde{\omega}, \frac{t}{2}\right). \tag{2.43}$$

Using the relation $J = \frac{1}{2}[H^{(1)} + H^{(2)}]$, $u^0(t)$ can be expressed in terms of $u^{\rm out}(t)$ as

$$\begin{aligned} u^0(t) &= \sqrt{\frac{\chi}{2\sinh 2\pi\tilde{\omega}}}\left[e^{\pi\tilde{\omega}+i\frac{\pi}{4}}u^{\rm out}(t) + e^{-\pi\tilde{\omega}-i\frac{\pi}{4}}u^{out*}(t)\right] \\ &= a^* u^{\rm out}(t) - b u^{out*}(t) \end{aligned} \tag{2.44}$$

and in terms of $u^{\rm in}(t)$ as

$$\begin{aligned} u^0(t) &= \tfrac{1}{\sqrt{2\chi\sinh 2\pi\tilde{\omega}}}\left[e^{\pi\tilde{\omega}-i\frac{\pi}{4}}u^{\rm in}(t) + e^{-\pi\tilde{\omega}+i\frac{\pi}{4}}u^{in*}(t)\right] \\ &= a u^{\rm in}(t) - b^* u^{in*}(t)\,. \end{aligned} \tag{2.45}$$

The Bogolubov coefficients relating the $|0\rangle_s$ vacuum to the $|in\rangle_s$ and $|out\rangle_s$ vacua are given by

$$a = \frac{e^{\pi\tilde{\omega}-i\frac{\pi}{4}}}{\sqrt{2\chi\sinh 2\pi\tilde{\omega}}}\,, \qquad b = -e^{-\pi\tilde{\omega}-i\frac{\pi}{4}}\sqrt{\frac{\chi}{2\sinh 2\pi\tilde{\omega}}}\,. \tag{2.46}$$

In the limit $\omega, \omega/\lambda \gg 1$, the s-brane wave functions may be understood in terms of a flux matching procedure.[9] In this limit the Klein-Gordon equation (2.26) describes brane creation (2.10) in the far past and brane decay in the far future, separated by a long region near $t = 0$ where the interaction is negligible. Approximate solutions to (2.26) may be found by matching wave functions of brane creation with wave functions of brane decay across the region $t \sim 0$. In particular, $u^{\rm in}$ and $u^{\rm out}$ look like the half s-brane solutions $u^{\rm in}_-$ and $u^{\rm out}_+$ in the past and future, respectively. The wave function u^0 looks like an ordinary plane wave solution in the interior of the s-brane, $u^{\rm out}_-$ in the far past and $u^{\rm in}_+$ in far future. This may be verified by noting that to lowest order in $1/\omega^2$ the Bogolubov coefficients (2.46) are given by (2.16).

[9] A similar procedure was used to study scattering of scalar fields by a D3 brane in [31].

For transitions to and from the $|0\rangle_s$ vacuum, we find that the particle creation rate is governed by the "thermal" factors:

$$\begin{aligned}
\gamma_{0\to in} &= \frac{b}{a^*} = ie^{-2\pi\tilde{\omega}}\,, & \gamma_{in\to 0} &= \frac{-b}{a} = e^{-2\pi\tilde{\omega}-i\tilde{\theta}} \\
\gamma_{0\to out} &= \frac{b^*}{a} = -ie^{-2\pi\tilde{\omega}}\,, & \gamma_{out\to 0} &= \frac{-b^*}{a^*} = e^{-2\pi\tilde{\omega}+i\tilde{\theta}}\,,
\end{aligned} \tag{2.47}$$

where $e^{-i\tilde{\theta}} = \chi$. In the limit of large ω and $\omega/\lambda \gg 1$, $\tilde{\omega}, \tilde{\theta} \to \omega, \theta$ and we find the same particle creation rates as for the half s-branes. We see that the particle occupation numbers in the far past and future are both thermal at temperature $T = 1/4\pi$. The $|0\rangle_s$ vacuum is special in this respect.

The $|0\rangle_s$ state is also special because it is time reversal invariant and is the natural state defined by analytic continuation from euclidean space. This latter property can be readily seen from the fact that it is the unique state whose modes are bounded on the positive euclidean axis, as described in section 2.2. It is also the only state whose positive and negative frequency modes do not mix under imaginary time translation $t \to t + 2\pi i$. A general solution of the wave equation has the form

$$u = AJ\left(-2i\tilde{\omega}, \frac{t}{2}\right) + BJ\left(-2i\tilde{\omega}, -\frac{t}{2}\right). \tag{2.48}$$

For the $|0\rangle$ vacuum B vanishes, and the solution transforms as

$$u^0(t + 2\pi i) = e^{2\pi\tilde{\omega}}\, u^0(t) \tag{2.49}$$

under imaginary time translation. In all other states both A and B are non-zero, so this procedure mixes u with u^*. When ω is large and $\omega/\lambda \gg 1$, $\tilde{\omega}$ approaches ω and (2.49) is the usual imaginary time translation condition exhibited by plane waves in flat space. Moreover, in our choice of solutions (2.43) we have chosen the phase of A so that u^0 has the usual time reversal behavior

$$u^0(-t) = u^{0*}(t)\,. \tag{2.50}$$

2.6 More vacua

The wave equation (2.26) is second order, so for any given momentum $\vec{p}$ there is a one complex parameter family of normalized, positive frequency solutions $u(t)$. We can write these solutions as linear combinations

$$u^\alpha = \frac{1}{\sqrt{1 - e^{\alpha+\alpha^*}}}\left(u^0 + e^\alpha u^{0*}\right) \tag{2.51}$$

where α is a function of $\vec{p}$ with negative real part. This choice of modes defines a state $|\alpha\rangle$ for every such function $\alpha(\vec{p})$. The state is invariant under spatial translations when α is a function of p only. This is a very large family of vacua, which includes the $|in\rangle_s$, $|0\rangle_s$ and $|out\rangle_s$ states. Of course, one can define families of vacua for brane creation and brane decay as well.

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In order to constrain α further, one needs to demand some other symmetry. One such symmetry is time reversal invariance — this restricts us to states such as $|0\rangle_s$ and $\frac{1}{2}(|in\rangle_s + |out\rangle_s)$. It is also natural to demand that the short distance structure of the vacuum be the same as the $|0\rangle_s$ vacuum. This restricts $\alpha(p)$ to vanish sufficiently quickly at large p.

3. S-brane thermodynamics

In this section we study the thermal properties of s-branes. In the first subsection we will consider the response of a monopole Unruh detector coupled to ϕ. We will show that in the $|in\rangle_-$ and $|out\rangle_+$ vacua the detector response is thermal, but that in other states the detector response is non-thermal. Next, we will demonstrate that in the $|in\rangle_-$ vacuum the correlators of the theory become exactly thermal up to corrections that vanish in the far future. Likewise, the $|out\rangle_+$ vacuum looks thermal in the far past. Similar results pertain to the full s-brane, but we will not work out the details here.

3.1 S-branes at finite temperature

The s-branes described by (2.2)–(2.4) are highly time-dependent configurations. Temperature is an equilibrium (or at best adiabatic) concept, so it usually does not make sense to put a time dependent configuration at finite temperature unless the inverse temperature is much lower than the scale of time variation. Thus it would seem to be impossible to study s-branes at string-scale temperatures. However, certain special properties of s-branes make this possible, as we will now demonstrate for a scalar field obeying (2.6). In section 5 we will see that the finite temperature states in string theory have a natural boundary state description.

Let us first consider the construction of Green functions by analytic continuation from euclidean space. In euclidean space, parameterized by $\tau = it$, the formulae (2.2)–(2.4) for $m^2(\tau)$ have the special property that they are periodic under $\tau \to \tau + 2\pi$. It is therefore possible to identify euclidean space so that $\tau = \tau + 2\pi n$ for any integer n, and compute the Green function on the identified space. After continuing back to Minkowski space, the resulting Green function $G_{2\pi n}(\vec{x}, t; \vec{x}', t')$ will be periodic under $t \to t + 2\pi i n$ and $t' \to t' + 2\pi i n$. This is a thermal Green function at temperature $T = \frac{1}{2\pi n}$.

The thermal Green function so obtained can also be understood as a two point function in a certain mixed state. We will consider the case of brane decay, although a similar discussion will apply to brane creation and to the full s-brane. For $t, t' \to -\infty$, $G_{2\pi n}(\vec{x}, t; \vec{x}', t')$ is clearly the usual thermal Feynman Green function which is given by (suppressing the time-ordering)

$$\begin{aligned} G_{2\pi n}(\vec{x}, t; \vec{x}', t') &= \sum_j e^{-2\pi n E_j} \langle E_j | \phi(\vec{x}', t') \phi(\vec{x}, t) | E_j \rangle \\ &= \mathrm{Tr} \rho_n \phi(\vec{x}', t') \phi(\vec{x}, t) \, . \end{aligned} \tag{3.1}$$

Here we have defined the density matrix

$$\rho_n = C_n e^{-2\pi n H(-\infty)} \tag{3.2}$$

where C_n is a normalization constant and the time dependent hamiltonian is

$$H(t) = \int_\Sigma d^d x((\dot{\phi})^2 + (\nabla\phi)^2 + e^{2t}\phi^2) + N\,. \tag{3.3}$$

In this expression, N is a (time-independent) normal ordering constant and ϕ is to be expanded in terms of time-independent creation and annihilation operators as

$$\phi = \sum_{\vec{p}} (\psi^{\rm in}_{\vec{p}} a^{\rm in}_{\vec{p}} + \psi^{in*}_{\vec{p}} a^{in\dagger}_{\vec{p}}) = \sum_{\vec{p}} (\psi^{\rm out}_{\vec{p}} a^{\rm out}_{\vec{p}} + \psi^{out*}_{\vec{p}} a^{out\dagger}_{\vec{p}})\,. \tag{3.4}$$

The hamiltonian $H(t)$ obeys

$$i[H(t), \phi(t)] = \partial_t \phi(t)\,, \qquad i[H(t), \partial_t\phi(t)] = \partial_t^2 \phi(t) \tag{3.5}$$

so that the path-ordered operator

$$U(t_2, t_1) = P\left[e^{i\int_{t_1}^{t_2} H(t)dt}\right] \tag{3.6}$$

generates time evolution on ϕ

$$\phi(\vec{x}, t_2) = U(t_2, t_1)\phi(\vec{x}, t_1)U(t_1, t_2)\,. \tag{3.7}$$

However, because of the explicit time dependence in (3.3), U does not generate evolution of $H(t)$.

In fact (3.1) turns out to be the desired Green function even for finite t, t', and (3.2) can be viewed as the exact Heisenberg-picture density matrix. First, it can be shown from properties of Bessel functions that

$$\begin{aligned} \phi(\vec{x}, t + 2\pi i n) &= e^{-2\pi n H(-\infty)}\phi(\vec{x}, t)e^{2\pi n H(-\infty)}\,, \\ \partial_t\phi(\vec{x}, t + 2\pi i n) &= e^{-2\pi n H(-\infty)}\partial_t\phi(\vec{x}, t)e^{2\pi n H(-\infty)}\,. \end{aligned} \tag{3.8}$$

Note that shifts in $t \to t + 2\pi i$ are generated by $H(-\infty)$ for *any* t. This implies

$$U(t, t + 2\pi i n) = U(-\infty, -\infty + 2\pi i n) = \frac{\rho_n}{C_n}\,. \tag{3.9}$$

It then follows that (3.1) obeys

$$G_{2\pi n}(\vec{x}, t + 2\pi i n; \vec{x}', t') = G_{2\pi n}(\vec{x}, t; \vec{x}', t' - 2\pi i n) = G_{2\pi n}(\vec{x}', t'; \vec{x}, t)\,, \tag{3.10}$$

as expected for a thermal Green function.

It is possible to define a time-dependent Schroedinger picture density matrix

$$\rho_n(t) = C_n U(-\infty, t + 2\pi i n)U(t, -\infty)\,. \tag{3.11}$$

One can then compute correlators at equal (finite) times by inserting the Schroedinger-picture operators $\phi(\vec{x}, -\infty)$:

$$G_{2\pi n}(\vec{x}, t; \vec{x}', t) = {\rm Tr}\rho_n(t)\phi(\vec{x}, -\infty)\phi(\vec{x}', -\infty)\,. \tag{3.12}$$

The Heisenberg density matrix is related to the Schrodinger density matrix by

$$\rho_n = \rho_n(-\infty)\,. \tag{3.13}$$

We note that the thermal density matrix ρ_n can not be exactly identified with any of the vacua of the preceding section, all of which are pure states by construction. Despite this fact we will see below that in some cases it becomes difficult to distinguish pure and mixed states.

To summarize, we have seen that, despite the time dependence of the background, it is mathematically possible to define mixed states which approaches the standard thermal vacuum at past infinity while retaining the thermal periodicity at all times.

3.2 Unruh detection

In the previous subsection, a sequence of mixed thermal states were described with temperatures $T = 1/2\pi n$. On the other hand, in the previous section we saw that various s-brane vacua — despite being pure states — have some mysterious "thermal" behavior. In this subsection we will clarify this by showing that a particle detector in these vacua will respond as if it is in a mixed thermal state with temperature $T = 1/4\pi$. In the next subsection we will show that the pure state correlators can asymptotically approach those of the thermal state.

Let us imagine coupling some detector to the field ϕ via a monopole interaction term $\int dt\ O(t)\phi(t)$. Here $O(t)$ is a hermitian operator that acts on the Hilbert space of the detector, which we will assume is spanned by some discrete, non-degenerate set of energy eigenstates $|E_i\rangle$. We also assume that the detector is stationary and that the detector hamiltonian is time independent. We can now calculate the probability that the detector will jump from a state with energy E_i to one with energy E_j. To first order in perturbation theory it is (a recent discussion appears in section 3.2 of [32])

$$P_{i\to j} = |\langle E_i|O(0)|E_j\rangle|^2 \int dt \int dt' e^{-i\Delta E(t-t')} G(t,t') \tag{3.14}$$

where $\Delta E = E_i - E_j$ and $G(t,t')$ is the Wightman two point function of the scalar field in a particular vacuum state. The detector response is thermal at temperature T if the probability amplitudes (3.14) obey the detailed balance condition

$$\frac{P_{i\to j}}{P_{j\to i}} = e^{-\Delta E/T}\,. \tag{3.15}$$

For time translation invariant theories the green function depends only on $t - t'$ and the double integral (3.14) is infinite. One can then factor out the $\int d(t+t')$ to get a finite expression for the transition rate per unit time $\dot{P}_{i\to j}$. For s-branes the Green function $G(t,t')$ is not time translation invariant and the calculation is more complicated.

In fact, for the full s-brane expression (3.14) — the *total* probability integrated over all time — is finite. This is because the Green function $G(x,y)$ solves the wave equation in both arguments, so that as $t \to \pm\infty$, $G(t,t') \sim e^{\mp t/4}$ vanishes rapidly. This holds for both t and t', so the double integral (3.14) converges. This is true for any vacuum state of the full s-brane. This behavior has a natural physical interpretation: in the far past and far future the open string states become very massive and cannot couple to the detector.

For brane creation, the Green function $G(t, t')$ does not fall off exponentially as $t \to +\infty$, so that the integrated probability amplitudes (3.14) are infinite. At this point one approach is to consider the transition rates $\dot{P}_{i\to j}(t)$, which are finite and time dependent. We will take a different point of view, however, and show directly that the ratio (3.15) converges to a finite (and interesting) answer.

In order to evaluate (3.14), we rewrite the Green function in a particular vacuum as

$$\begin{aligned} G(x^0, \vec{x}; y^0, \vec{y}) &= \langle|\phi(x^0, \vec{x})\phi(y^0, \vec{y})|\rangle \\ &= \int d^{d-1}p\ e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}u^*(x^0)u(y^0) \end{aligned} \tag{3.16}$$

where u are the (p-dependent) positive frequency modes. Then (3.14) becomes

$$P_{i\to j} = \int d^{d-1}\vec{p}\,|\tilde{u}(E_i - E_j)|^2 \tag{3.17}$$

where $\tilde{u}$ are the Fourier transformed modes. For the $|out\rangle_+$ vacuum of brane decay we can evaluate the Fourier integral[10]

$$\begin{aligned} \tilde{u}_+^{\rm out}(E) &= \int e^{-iEt}u_+^{\rm out}(t)\ dt \\ &= \sqrt{\frac{i}{8\pi}}e^{-\pi E}\lambda^{iE}\Gamma(-i(\omega+E))\Gamma(i(\omega-E))\,. \end{aligned} \tag{3.18}$$

This has the form

$$\tilde{u}_+^{\rm out}(E) = i^{1/2}e^{-\pi E}x(E) \tag{3.19}$$

where $x(-E) = x^*(E)$. The norm is

$$|\tilde{u}_+^{\rm out}(E)|^2 = e^{-2\pi E}\,[xx^*]\,, \tag{3.20}$$

where the term in the square brackets is invariant under $E \to -E$. From this fact and expression (3.17) it follows that the transition probabilities in the $|out\rangle_+$ vacuum satisfy the detailed balance condition (3.15) with characteristic temperature $T_H = \frac{1}{4\pi}$.[11] The $|in\rangle_-$ vacuum for brane creation is related to the $|out\rangle_+$ vacuum for brane decay by time reversal, so it has identical transition probabilities $P_{i\to j}$ and thermal detector response.

In fact, the $|in\rangle_-$ and $|out\rangle_+$ vacua are very special in this respect. For example, we could calculate the transition probabilities in the $|in\rangle_+$ vacuum by inverting the Bogolubov relation (2.15). This gives

$$\begin{aligned} \tilde{u}_p^0(E) &= c^*\tilde{u}_p^{\rm out}(E) - d\tilde{u}_p^{\rm out *}(-E) \\ &= -2i^{3/2}de^{\pi\omega}\cosh\pi(\omega - E)x\,. \end{aligned} \tag{3.21}$$

[10] As usual, the integral converges only after we deform the contour to give E a small positive imaginary part, $E \to E + i\epsilon$.

[11] As mentioned above, these transition probabilities are infinite, so in evaluating (3.14) it is necessary to regulate the divergent momentum integrals by, e.g. imposing some cutoff. In evaluating the ratio (3.15) one can safely take this regulator to infinity.

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The norm is

$$|\tilde{u}_p^0(E)|^2 = \cosh^2 \pi(\omega - E) \left[4|d|^2 e^{2\pi\omega} x x^*\right] \tag{3.22}$$

where again the quantity in the square brackets is invariant under $E \to -E$. In this case it is clear that the transition probabilities are not thermal. It seems likely that for *any* vacuum state that is not $|out\rangle_+$ or $|in\rangle_-$, the Bogolubov transformation will give terms proportional to $e^{\pi E}$ so that the detector response is non-thermal.

3.3 Thermal correlators

We will now demonstrate that the correlators of the $|in\rangle_-$ vacuum are asymptotically thermal in the far future. The theory is free, so it suffices to consider the two point function $G(x,y)$. For brane creation, we have

$$u_-^{\rm in} = a(u_-^{\rm out} - e^{-2\pi\omega + i\theta} u_-^{\rm out *}) \tag{3.23}$$

so the $|in\rangle_-$ Green function is

$$\begin{aligned} G_-^{\rm in}(x,y) &= \int d^{d-1}\vec{p}\; u^{\rm in}(x) u^{\rm in *}(y) \\ &= \int \frac{d^{d-1}\vec{p}}{1 - e^{-4\pi\omega}} e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \Big[(u_0(t) u_0^*(t') + e^{-4\pi\omega} u_0^*(t) u_0(t')) \\ &\qquad + e^{-2\pi\omega} (e^{i\theta} u_0(t) u_0(t') + \text{c.c.}) \Big] . \end{aligned} \tag{3.24}$$

In the far future, $u_-^{\rm out}$ approaches a positive frequency plane wave plus corrections exponentially small in $t + t'$. In this limit the term on the second line of (3.24) becomes a function of $t - t'$ only, and approaches the usual (constant mass) thermal Green function at temperature $T_H = 1/4\pi$

$$G^T(x,y) = \int \frac{d^{d-1}\vec{p}}{2\omega} \left(\frac{e^{i\vec{p}\cdot(\vec{x}-\vec{y}) - i\omega(t-t')}}{1 - e^{-4\pi\omega}} - \frac{e^{i\vec{p}\cdot(\vec{x}-\vec{y}) + i\omega(t-t')}}{1 - e^{4\pi\omega}} \right) . \tag{3.25}$$

In the far future the third line of (3.24) depends on $t + t'$ rather than $t - t'$, and gives a contribution to the Green function

$$\int \frac{d^{d-1}\vec{p}}{2\omega} \frac{2}{\sinh 2\pi\omega} \left(e^{i\theta} e^{i\vec{p}\cdot(\vec{x}-\vec{y}) - i\omega(t+t')} + \text{c.c.} \right) \tag{3.26}$$

plus exponentially small corrections. In the limit $t, t' \to +\infty$ this contribution vanishes as $(t+t')^{-(d-1)/2}$. In fact, when $\omega^2 = p^2$ (i.e. the field becomes massless in the far future) the integral (3.26) can be converted into a contour integral and shown to vanish exponentially in $t + t'$.

We conclude that in the far future the pure state $|in\rangle_-$ correlators become thermal plus asymptotically vanishing corrections. Likewise the $|out\rangle_+$ correlators become thermal in the far past.

4. Long-distance s-brane effective field theory

The dynamics of ordinary D-branes are described at low energies by a long-distance effective field theory. This field theory is of much interest in the understanding of holographic bulk-brane duality. One would like to know if there is a similar long-distance field theory for s-branes, which would be a candidate holographic dual for an appropriate bulk string cosmology.

The first task is to define the notion of an effective field theory for branes with a spacelike orientation. This is best understood in terms of correlators. We define the long-distance effective field theory on the p+1-dimensional sp-brane as the p+1-dimensional euclidean field theory that reproduces the long-distance correlators. To be specific, let us consider an s2-brane which is real codimension one in four spacetime dimensions. A massless field confined to the s2-brane should have a correlator that falls off like $\frac{1}{r}$. This is quite distinct from massless correlators at spacelike-separated points in the ambient four-dimensional spacetime, which fall off like $\frac{1}{r^2}$.

Consider a scalar on the full s2-brane with four-dimensional wave equation:

$$\partial^\mu \partial_\mu \phi - (2\lambda \cosh t + \omega^2)\phi = 0\,, \tag{4.1}$$

as in (2.26). In the far future and far past, ϕ is very massive, and correlators fall off exponentially with the spatial separation. Hence if there are any massless excitations they will be confined to the s2-brane world-volume near $t = 0$ where ϕ is light.

In trying to compute the equal-time correlators near $t = 0$, we immediately encounter a puzzle. Such correlators depend on the choice of quantum state for the field ϕ. Indeed, given any set of equal time correlators $\Delta(\vec{x}, \vec{y})$ there exists a quantum state $\Psi[\phi] = exp[-\frac{1}{4}\int\int \phi(\vec{x})\Delta^{-1}(\vec{x}, \vec{y})\phi(\vec{y})]$ that reproduces them.

In order to determine the long-distance effective field theory we must therefore first specify a quantum state. One possibility is to take the $|0\rangle_s$ state which has no particle flux at $t = 0$. Spacelike correlators at $t = 0$ in this state fall off exactly as they would in Minkowski space, *i.e.* as $1/r^2$. So this does not lead to a low energy effective field theory confined to the s-brane world-volume. As we have seen, a natural state for an s-brane is the thermal state at temperature $T = 1/2\pi n$. Such thermal states plausibly approximate the quantum states of open strings on an s-brane created from incoming closed string excitations. The $t = 0$ correlators in these states indeed fall off as $1/r$, indicating massless modes are confined to the s-brane. This can be most easily seen from the euclidean construction of the correlators on $R^3 \times S^1$. At distances large compared to the radius of the S^1 (i.e. large compared to the inverse temperature) there is an effective compactification from four to three (euclidean) dimensions, and so the effective correlators are three-dimensional. Massless modes of the three dimensional effective theory arise as usual from compactification zero modes. From the minkowskian perspective, the mixed thermal state has excited components which carry more spatial correlations than the vacuum.

The euclidean zero mode equation following from (4.1) is

$$\partial_\tau^2 \phi - (2\lambda \cos \tau + \omega^2)\phi = 0\,, \tag{4.2}$$

where $\tau \sim \tau + 2\pi n$. Whether or not there is a zero mode for a given temperature depends on the precise value of the mass parameter ω^2 (4.2), which has to be fine-tuned to get an exact zero mode. This special value might arise as a consequence of Goldstone's theorem or other symmetry considerations.

In conclusion, the s2-brane has a naturally associated three-dimensional euclidean effective field theory given roughly by the high-temperature limit of the theory on the four-dimensional unstable brane. The full determination of such an effective theory for stringy s-branes is beyond the scope of the present work, although a few preliminary comments are made in our concluding discussion.

5. Thermal boundary states

We have seen that natural quantum states for s-branes are mixed thermal states at temperature $T = 1/2\pi n$. In this section we will construct the exact CFT boundary state whose worldsheet correlators give the thermal spacetime Green functions, generalizing the zero-temperature construction of [3]. Ghost and spacelike components of the boundary state are suppressed, but are similar to those in [3].

5.1 Zero modes and winding sectors

The boundary state $|B\rangle$ for the conformally invariant boundary Sine-Gordon theory

$$S = \frac{1}{2\pi}\int_\Sigma \partial\phi\bar{\partial}\phi + \frac{\lambda}{2}\int_{\partial\Sigma}(e^{i\phi} + e^{-i\phi}) \tag{5.1}$$

was found using the bulk SU(2) current algebra in [26] (see also [33, 34, 35]). Here ϕ is euclidean time, which will later be analytically continued to the lorentzian world sheet field $X^0 = i\phi$. The boundary state corresponding to temperature $T = 1/2\pi n$ arises when one identifies

$$\phi \sim \phi + 2\pi n\,. \tag{5.2}$$

This identification restricts the left and right moving momenta to be

$$(p_L, p_R) = \left(\frac{p}{n} + wn, \frac{p}{n} - wn\right) \tag{5.3}$$

where p and w are integers. In this case the thermal boundary state is simply

$$|B_n\rangle = P_n\ e^{2\pi i\lambda J_1}|N\rangle. \tag{5.4}$$

Here

$$|N\rangle = 2^{-1/4}\sum_{j,m}|j;m,-m\rangle\rangle \tag{5.5}$$

is the standard SU(2) Neumann boundary state and $|j;m,-m\rangle\rangle$ is the Ishibashi state associated with the SU(2) primary field $|j;m,-m\rangle$. The SU(2) rotation J_1 acts only on right-movers, and P_n is the projection operator onto the allowed sub-lattice defined by (5.3). For $n = 1$, P_1 is the identity. In the non-compact case, $n \to \infty$ and P_∞ projects onto $p_L = p_R$.

Let us now consider the part of the boundary state which involves no oscillators. There are four such terms for every j, namely $|j;\pm j,\pm j\rangle$. Now, since

$$\langle j;j,j|P_n = \langle j;j,j|P_\infty\,, \qquad \langle j;-j,-j|P_n = \langle j;-j,-j|P_\infty \tag{5.6}$$

for any n, the $p_L = p_R$ components of the state are the same as in [3]. For real λ they may be written

$$[1+2\sum_{j\neq 0}(-\sin(\pi\lambda))^{2j}\cos(2j\phi(0))]|0\rangle = \frac{\cos^2(\pi\lambda)}{1+\sin^2(\pi\lambda)+2\sin(\pi\lambda)\cos\phi(0)}|0\rangle\,. \tag{5.7}$$

For finite n, there are also terms with $p_L = -p_R = 2nj \neq 0$. These are related to the $p_L = p_R$ terms by the rotation $e^{i\pi J_1}$, which corresponds to a shift of λ by $\frac{1}{2}$. Hence in addition to (5.7), $|B_n\rangle$ has a winding-sector component

$$2\sum_{j\neq 0}(\cos\pi\lambda)^{2nj}\cos(2nj\tilde{\phi}(0))|0\rangle = \frac{2\cos^n(\pi\lambda)\cos(n\tilde{\phi}(0)) - 2\cos^{2n}(\pi\lambda)}{1+\cos^{2n}(\pi\lambda)-2\cos^n(\pi\lambda)\cos(n\tilde{\phi}(0))}|0\rangle\,. \tag{5.8}$$

In this expression $\tilde{\phi}(z,\bar{z}) = \frac{1}{2}(\phi(z)-\phi(\bar{z}))$ is the T-dual of ϕ.

The continuation $\phi \to -iX^0$ of these expressions to the timelike theory

$$S = -\frac{1}{2\pi}\int_\Sigma \partial X^0\bar{\partial}X^0 + \lambda\int_{\partial\Sigma}\cosh X^0 \tag{5.9}$$

is straightforward. The theory (5.9) contains the currents[12]

$$j_\pm(z) = e^{\pm X^0(z)}\,, \qquad j_3(z) = \frac{1}{2}\partial X^0(z)\,, \tag{5.10}$$

which generate the usual level one SU(2) current algebra. Note however that with the standard norm $X^{0\dagger} = X^0$, j_3 is anti-hermitian while $j^\pm$ are both hermitian. Nevertheless the charges

$$J_\pm = \oint\frac{dz}{2\pi i}j_\pm(z)\,, \qquad J_3 = \oint\frac{dz}{2\pi i}j_3(z)\,, \tag{5.11}$$

obey the usual commutation relations

$$[J_-,J_+] = -2J_3\,, \qquad [J_3,J_\pm] = \pm J_\pm\,. \tag{5.12}$$

States can therefore be characterized by their SU(2) representations. Under $\phi \to -iX$, $J_k \to J_k$ and hence $|j;m,m'\rangle\rangle \to |j;m,m'\rangle\rangle$. Therefore the Sine Gordon boundary state written in the form (5.4) can also be viewed as a boundary state for the timelike theory (5.9). Expressions (5.7) and (5.8) become

$$\left[1+2\sum_{j\neq 0}(-\sin(\pi\lambda))^{2j}\cosh(2jX^0(0))\right]|0\rangle = \frac{\cos^2(\pi\lambda)}{1+\sin^2(\pi\lambda)+2\sin(\pi\lambda)\cosh X^0(0)}|0\rangle \tag{5.13}$$

and

$$\sum_{j\neq 0}(\cos\pi\lambda)^{2nj}\cosh(2nj\tilde{X}^0(0))|0\rangle = \frac{2\cos^n(\pi\lambda)\cosh(n\tilde{X}^0(0)) - 2\cos^{2n}(\pi\lambda)}{1+\cos^{2n}(\pi\lambda)-2\cos^n(\pi\lambda)\cosh(n\tilde{X}^0(0))}|0\rangle\,. \tag{5.14}$$

Here $\tilde{X}^0(z,\bar{z}) = \frac{1}{2}(X^0(z)-X^0(\bar{z}))$.

[12] In our conventions $X(z,\bar{z}) = \frac{1}{2}(X(z)+X(\bar{z}))$, $X(z)X(w) \sim 2\ln(z-w)$ and $\alpha' = 1$.

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5.2 The superstring

In this subsection we sketch the finite-temperature generalization for the superstring of the zero-temperature boundary state of [4]. For the superstring, instead of the boundary interaction (2.1), one has

$$\lambda \int d\tau \psi^0 \sinh \frac{X^0}{\sqrt{2}} \otimes \sigma_1 \,. \tag{5.15}$$

We follow the notation of [2, 4, 14] in which σ_1 acts on Chan-Paton factors, and (5.15) arises from a tachyon field proportional to $\cosh \frac{X^0}{\sqrt{2}}$. After integrating out ψ^0 one obtains a boundary interaction of the form (2.4). This interaction is invariant under

$$X^0 \to X^0 + 2\sqrt{2}\pi i \,. \tag{5.16}$$

We can therefore consider thermal boundary states at temperatures

$$T = \frac{1}{2\sqrt{2}\pi n} \,. \tag{5.17}$$

This corresponds to the superstring Hagedorn temperature $T_H = \frac{1}{2\sqrt{2}\pi}$ for the minimal value $n = 1$, rather than the $n = 2$ we encountered in the bosonic string. The bosonic zero mode part of the boundary state with no winding is then

$$\frac{1 - \sin^4(\pi\lambda)}{1 + \sin^4(\pi\lambda) + 2\sin^2(\pi\lambda)\cosh(\sqrt{2}X^0(0))}|0\rangle \,. \tag{5.18}$$

The winding component is

$$\frac{2\cos^{2n}(\pi\lambda)\cosh(\sqrt{2}n\tilde{X}^0(0)) - 2\cos^{4n}(\pi\lambda)}{1 + \cos^{4n}(\pi\lambda) - 2\cos^{2n}(\pi\lambda)\cosh(\sqrt{2}n\tilde{X}^0(0))}|0\rangle \,. \tag{5.19}$$

These components are similar to those of the bosonic string, with the replacement $\sin\pi\lambda \to \sin^2\pi\lambda$ and the factors of $\sqrt{2}$. In computing the fermionic components, twisted boundary conditions around the thermal circle must be taken into account.

6. The $\lambda = \pm 1/2$ sD-brane limit

In this section we will consider the very interesting limits $\lambda \to \pm 1/2$. In [14] it was shown that in a certain $\lambda \to -1/2$ limit, the general bosonic s-brane boundary state (5.4) (at zero temperature) reduces to the boundary state of [1] which imposes a Dirichlet boundary condition $X^0 = 0$ in the time direction. In other words $\lambda = -1/2$ is an sD-brane. The relation to sD-branes follows immediately from the fact that the $\lambda = \pm 1/2$ boundary interaction is an SU(2) rotation by π which transforms a Neumann boundary state into a Dirichlet state.

On the other hand, in [3] it was shown that in a certain $\lambda \to 1/2$ limit, the general s-brane boundary state in some sense reduces to nothing — i.e. in this limit there is no brane present at all.[13] In fact, we will see that the limiting closed string configuration is

[13] As can be seen from (5.18), the superstring case for both $\lambda \to 1/2$ and $\lambda \to -1/2$ is qualitatively similar to the $\lambda \to 1/2$ bosonic case.

not unambiguously determined from the boundary state for any value of λ. Additional boundary conditions on the fields are needed. In general the limit is not the trivial one described in [3], but rather is a very special type of s-brane configuration described by spacelike Dirichlet branes located on the imaginary time axis.

In the next subsection, we will describe this ambiguity in the limit $\lambda \to \frac{1}{2}$. In section 6.2 we will derive the linearized RR-field sourced by an sD-brane and see that it carries a non-trivial s-charge for all λ. In 6.3 the long range graviton and dilaton fields of an s-brane are computed. In 6.4 we determine the force between an ordinary D-brane and a $\lambda = 1/2$ sD-brane from a computation of the annulus diagram. in 6.5 the calculation is generalized to finite temperature. Finally, in section 6.6 we discuss the relation between s-branes and D-instantons.

6.1 The classical closed string field

Consider the state

$$|C\rangle = \frac{1}{L_0 + \bar{L}_0}|B\rangle \,. \tag{6.1}$$

This can be viewed as a quantum state of a single closed string. Alternately, since the states in the Fock basis of the single closed string are identified as spacetime components of the classical string field, $|C\rangle$ can be viewed as a classical string field configuration. By construction it obeys

$$(L_0 + \bar{L}_0)|C\rangle = |B\rangle \,. \tag{6.2}$$

These are the linearized spacetime wave equations with sources for the components of the classical string field. The source is the boundary state $|B\rangle$ whose support is confined to the brane. Hence we conclude that $|C\rangle$ is the linearized classical closed string field sourced by the brane.

For ordinary static supersymmetric D-branes, it has been explicitly verified that $|C\rangle$ as defined in (6.1) reproduces the linearized dilaton, metric and RR fields sourced by the brane [36]. When the brane is static, the Fock component states in $|B\rangle$ carry no p^0, and all of its components are off-shell. A unique static $|C\rangle$ may then be determined from (6.1). Time-dependent boundary states differ crucially in this regard. They have components with non-vanishing p^0 which correspond to on-shell closed string states and hence are annihilated by $L_0 + \bar{L}_0$. It follows that $L_0 + \bar{L}_0$ is not invertible and $|C\rangle$ is not unambiguously determined from $|B\rangle$. This is just the usual problem of specifying the homogenous part of the solution emanating from a time-dependent source. Some additional boundary conditions must be specified.[14]

For general λ, the Fock components of the string field wave equation (6.2) are of the form

$$\begin{aligned}\eta^{ab}\partial_a\partial_b\phi(\vec{x},t) &= \delta^{25-p}(\vec{x})\frac{\cos^2(\pi\lambda)}{1+\sin^2(\pi\lambda)+2\sin(\pi\lambda)\cosh t} \\ &= \delta^{25-p}(\vec{x})\left(\frac{1}{1+e^t\sin\pi\lambda}+\frac{1}{1+e^{-t}\sin\pi\lambda}-1\right).\end{aligned} \tag{6.3}$$

[14] We note that this closed string ambiguity remains even after we have fixed the quantum state of the open strings on the brane.

This follows from (5.13) after replacing the operator X^0 by its eigenvalue t. Here $\vec{x}$ are the transverse spatial dimensions, and longitudinal spatial dimensions are suppressed. For notational simplicity we henceforth specialize to $p = 22$, so that only four spacetime dimensions are relevant. For $\lambda = 1/2$ the wave equation (6.3) reduces to

$$\eta^{ab}\partial_a\partial_b\phi(\vec{x},t) = 2\pi i\delta^3(\vec{x}) \sum_{m=-\infty}^{\infty} \delta(t+\pi i+2m\pi i)\,. \tag{6.4}$$

Since the source on the right hand side vanishes for real t, an obvious solution of (6.4) for real t is simply

$$\phi(\vec{x},t) = 0\,. \tag{6.5}$$

This is the solution implicit in [3]. However, there is another solution which "knows" about the sources at imaginary t. Recall that the wave equation with a delta function source

$$\eta^{ab}\partial_a\partial_b\phi(\vec{x},t) = \delta^3(\vec{x})\delta(t-t_0) \tag{6.6}$$

is solved by Feynman propagator

$$\phi(\vec{x},t) = -\Delta_F(\vec{x},t;\vec{0},t_0) = \frac{i}{4\pi^2}\lim_{\epsilon\to 0}\frac{1}{(t-t_0)^2-r^2+i\epsilon}\,, \tag{6.7}$$

where $r^2 = \vec{x}^2$. Continuing this to imaginary t_0 we find that (6.3) is solved by

$$\phi(\vec{x},t) = -2\pi i \textstyle\sum_{m=-\infty}^{\infty} \Delta_F(\vec{x},t;\vec{0},\pi i+2\pi m i) = \displaystyle\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\frac{1}{r^2-(t-\pi i-2\pi m i)^2} \tag{6.8}$$

Since the denominator is non-vanishing for real (r,t), we have set ϵ to zero here. Performing the sum over m yields

$$\phi(\vec{x},t) = -\frac{1}{4\pi r}(\tanh\frac{r+t}{2} + \tanh\frac{r-t}{2})\,. \tag{6.9}$$

Note that at large t and fixed r this vanishes exponentially. On the other hand, at fixed time t this has the $1/r$ falloff at large r characteristic of a static source, yet it is nonsingular for all real r, t. With 25-p transverse dimensions we would find a characteristic $1/r^{23-p}$ falloff. Solutions of this general form were discussed in [1].

In section 6.3 we will compute the annulus diagram connecting an sD-brane and an ordinary D-brane using a straightforward adaptation of the standard string theory prescription [37], which involves a euclidean continuation of the worldsheet field X^0. Since this calculation gives a definite answer, it must contain an implicit prescription for inverting $L_0 + \bar{L}_0$. We shall find that at large separation the graviton falls off like $1/r^{23-p}$. This indicates that non-trivial solutions of the form (6.9) rather than the trivial solution (6.5) are implicit in this formulation of worldsheet string theory.

We note that at $\lambda = /$, the branes have no support at real t, and therefore there can be no on-shell open strings propagating at real time. Hence the problem [12] of a Hagedorn-like divergence in open string pair production disappears.

6.2 RR field strength and s-charge

In this subsection we will use (6.1) to determine the RR field sourced by an sD-brane in superstring theory. We will find that it carries one unit of "s-charge" defined below as an integral of the RR field strength over a complete spacelike surface.

In the presence of an s-brane described by the boundary deformation (5.15), the source for RR fields is proportional to [38]

$$\sin(\pi\lambda)\left[\frac{e^{X^0/\sqrt{2}}}{1+\sin^2(\pi\lambda)e^{\sqrt{2}X^0}}-\frac{e^{-X^0/\sqrt{2}}}{1+\sin^2(\pi\lambda)e^{-\sqrt{2}X^0}}\right] \tag{6.10}$$

up to an overall normalization, which can be determined as follows. If we analytically continue

$$\lambda\to -i\lambda\,,\qquad X^0\to X^0+\pi i/\sqrt{2}\,, \tag{6.11}$$

the boundary interaction (5.15) becomes $\lambda\psi^0\cosh X^0/\sqrt{2}$, which describes the tachyon rolling over the barrier. The corresponding s-brane should carry ± 1 unit of RR charge, with the sign depending on the sign of λ. The integral of the source is now $\sqrt{2}\pi\,\mathrm{sign}(\lambda)$, which determines the normalization factor to be $(\sqrt{2}\pi)^{-1}$ times the unit RR charge.

Let us now return to the case (5.15). After euclidean continuation $X^0\to i\phi$, the RR source can be written as

$$\frac{1}{\sqrt{2}\pi}\sin(\pi\lambda)\left[\sum_{n=0}^{\infty}(-1)^n\sin^{2n}(\pi\lambda)e^{i(2n+1)\phi/\sqrt{2}}-\sum_{n=0}^{\infty}(-1)^n\sin^{2n}(\pi\lambda)e^{-i(2n+1)\phi/\sqrt{2}}\right]. \tag{6.12}$$

When $\lambda = 1/2$, this is the source corresponding to an array of branes and anti-branes located along the euclidean time axis

$$-i\sqrt{2}\sum_{n=-\infty}^{\infty}(-1)^n\delta(\sqrt{2}\phi+\pi+2n\pi) \tag{6.13}$$

The wave equation is then

$$\partial^a\partial_a C_{9-p,\cdots,9}=\sqrt{2}\sum_{n=-\infty}^{\infty}(-1)^n\delta(\sqrt{2}X^0+i\pi+2\pi ni)\delta(\vec{r}_\perp)\,. \tag{6.14}$$

For simplicity, we will consider the case of an s5-brane in type-IIB theory, so that there are 4 transverse directions. As in the previous section, we can solve the wave equation in euclidean space and analytically continue back to find

$$\begin{aligned} C_{4\cdots 9} &= \frac{i}{2\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{(\sqrt{2}t+\pi i+2\pi ni)^2-2r^2}\\ &= \frac{1}{8\sqrt{2}\pi^2 r}\left[\frac{1}{\cosh\frac{r-t}{\sqrt{2}}}-\frac{1}{\cosh\frac{r+t}{\sqrt{2}}}\right]. \end{aligned} \tag{6.15}$$

The RR potential for sp-branes, $p\neq 5$, in type-IIB theory can be obtained from the above by, say, taking derivatives in r.

The s-brane with $\lambda = 1/2$ has no RR source located at real time, so the RR flux through any transverse spacelike 3-surface is conserved. The conserved charge is

$$Q_s = \int_{\Sigma_3} *dC\,. \tag{6.16}$$

If we take Σ_3 to be the plane located at $X^0 = t$ and extending in $X^{1,2,3}$ directions, then

$$(dC)_{04\cdots 9} = \frac{\partial}{\partial_t} C_{4\cdots 9} = \frac{1}{16\pi^2 r}\left[\frac{\sinh\frac{r-t}{\sqrt{2}}}{\cosh^2\frac{r-t}{\sqrt{2}}} + \frac{\sinh\frac{r+t}{\sqrt{2}}}{\cosh^2\frac{r+t}{\sqrt{2}}}\right] \tag{6.17}$$

yields

$$Q_s = \int_0^\infty dr 4\pi r^2 (dC)_{04\cdots 9} = \tfrac{1}{2}\,. \tag{6.18}$$

We conclude that the $\lambda = \frac{1}{2}$ closed string configuration carries half a unit of spacelike Ramond-Ramond charge.

6.3 Long-range graviton/dilaton fields

In this subsection we compute the long-distance Coulomb fields for the graviton and dilaton sourced by the sp-brane. We will restrict to the bosonic case, although the generalization to the superstring is straightforward.

The spatial components of the boundary state of an sp-brane are

$$|B\rangle_{\vec{X}} = \frac{T_{p+1}}{2}\delta^{24-p}(\vec{x}_\perp)\exp\left(-\sum_{n=1}^{\infty} S_{ij} a^i_{-n}\tilde{a}^i_{-n}\right)|0\rangle \tag{6.19}$$

where T_{p+1} is the tension of the D$(p+1)$-brane and S_{ij} $(1 \le i, j \le 25)$ is given by

$$S_{ij} = (\delta_{\alpha\beta}, -\delta_{ab}) \tag{6.20}$$

where α, β $(a, b\,)$ label the directions with Neumann (Dirichlet) boundary conditions.

The relevant parts of the time component of the boundary state are [3]

$$|B\rangle_{X^0} = f(X^0)|0\rangle + a^0_{-1}\tilde{a}^0_{-1} g(X^0)|0\rangle + \cdots \tag{6.21}$$

where

$$\begin{aligned} f(X^0) &= \frac{1}{1+e^{X^0}\sin\pi\lambda} + \frac{1}{1+e^{-X^0}\sin\pi\lambda} - 1, g(X^0) \\ &= 1 + \cos(2\pi\lambda) - f(X^0)\,, \end{aligned} \tag{6.22}$$

and a^0_{-1} is an oscillator in the expansion of X^0. Combining (6.19) and (6.21), we get the total source for graviton and dilaton

$$|B\rangle = \frac{T_{p+1}}{2}\delta^{24-p}(\vec{x}_\perp)\left[-S_{ij}a^i_{-1}\tilde{a}^j_{-1} f(X^0) + a^0_{-1}\tilde{a}^0_{-1} g(X^0)\right]|0\rangle + \cdots\,. \tag{6.23}$$

The massless part of the closed string field is then

$$|C\rangle = \frac{T_{p+1}}{2}V_{p+1}\int dt'\,\Delta(\vec{X}, X^0; 0, t')\left[-S_{ij}a^i_{-1}\tilde{a}^j_{-1} f(t') + a^0_{-1}\tilde{a}^0_{-1} g(t')\right]|0\rangle + \cdots \tag{6.24}$$

where V_{p+1} is the spatial volume of the s-brane. Our prescription for the t' integral employs the euclidean imaginary time axis. Defining

$$J^{\mu\nu}(k) = \langle 0; k | a_1^\mu \tilde{a}_1^\nu | C \rangle \tag{6.25}$$

one finds

$$\begin{aligned} J_{ij}(x) &= -\frac{T_{p+1}}{2} V_{p+1} S_{ij} \int dt' \Delta(\vec{x}, t; 0, t') f(t')\,, \\ J_{00}(x) &= \frac{T_{p+1}}{2} V_{p+1} \int dt' \Delta(\vec{x}, t; 0, t') g(t')\,. \end{aligned} \tag{6.26}$$

For simplicity let us first consider the case $p = 21$, where there are 4 transverse directions to the sp-brane. For $r = |\vec{x}| > |t|$,[15]

$$\begin{aligned} J_{ij}(\vec{x}, t) &= -\frac{T_{p+1} V_{p+1}}{2} \frac{S_{ij}}{4\pi r} \left[\frac{1}{1 + e^{t-r} \sin \pi\lambda} + \frac{1}{1 + e^{-t-r} \sin \pi\lambda} - 1 \right], \\ J_{00}(\vec{x}, t) &= -\frac{T_{p+1} V_{p+1}}{2} \frac{1}{4\pi r} \left[\frac{1}{1 + e^{t-r} \sin \pi\lambda} + \frac{1}{1 + e^{-t-r} \sin \pi\lambda} - 2 - \cos(2\pi\lambda) \right]. \end{aligned} \tag{6.27}$$

In the limit of large r, we have

$$J_{ij} \to -\frac{T_{p+1} V_{p+1}}{2} \frac{S_{ij}}{4\pi r}\,, \qquad J_{00} \to \frac{T_{p+1} V_{p+1}}{2} \frac{\cos(2\pi\lambda)}{4\pi r} \tag{6.28}$$

For general sp-branes

$$\begin{aligned} J_{ij} &\to -N_p \frac{S_{ij}}{r^{22-p}}\,, \qquad J_{00} \to N_p \frac{\cos(2\pi\lambda)}{r^{22-p}}\,, \\ N_p &= \frac{T_{p+1} V_{p+1}}{4} \pi^{\frac{p-24}{2}} \Gamma\left(\frac{24-p}{2} \right). \end{aligned} \tag{6.29}$$

We see that in the string frame the graviton and dilaton fields fall off like $1/r^{22-p}$. This is consistent with the results of the next subsection in which the force between an s-brane and a D-brane is computed.

6.4 The annulus diagram

In this subsection we compute the bosonic annulus diagram in the presence of an ordinary D(p+1)-brane and sp-brane with general λ. $\lambda = 0$ corresponds to two D(p+1)-branes while $\lambda = 1/2$ is an sDp-brane and a D(p+1)-brane. We will deduce the long range force from this computation and find that it is in agreement with the results of the previous section.

Let us denote the boundary state associated to the D(p+1)-brane by $|D_{p+1}\rangle$ and the boundary state associated to the sp-brane with coupling λ by $|s_p, \lambda\rangle$. They are factorized as the product of time components and spatial components as

$$\begin{aligned} |D_{p+1}\rangle &= |N\rangle^0 \otimes |N\rangle^{1,\cdots,p+1} \otimes |D\rangle^{p+2,\cdots,25} \otimes |\text{ghost}\rangle, \\ |s_p\rangle &= |B, \lambda\rangle \otimes |N\rangle^{1,\cdots,p+1} \otimes |D\rangle^{p+2,\cdots,25} \otimes |\text{ghost}\rangle\,. \end{aligned} \tag{6.30}$$

[15] The full solution is discontinuous (but obeys the wave equation) on the light cone for $\lambda < \frac{1}{2}$. The $\lambda = 0$ case corresponds to a D-brane with some radiation inside the light cone.

Here $|B,\lambda\rangle$ is the zero-temperature boundary state (as in [26, 3] and equation (5.4))

$$|B,\lambda\rangle = P_\infty e^{2\pi i\lambda J_1}|N\rangle, = 2^{-1/4}\sum_{j,m} D^j_{m,-m}(2\pi\lambda)|j,m,m\rangle\rangle. \qquad (6.31)$$

Here $J_1 = \cosh X^0$, P_∞ projects onto $p_L = p_R$ and $D^j_{m,m'}$ is the SU(2) representation matrix element

$$D^j_{m,m'}(2\pi\lambda) = \langle j,m|e^{i2\pi\lambda J_1}|j,m'\rangle\,. \qquad (6.32)$$

For the state $|j,m,m'\rangle\rangle$ the left and right momenta are related by $p_L = 2m', p_R = 2m$.

The annulus diagram connecting $|D_{p+1}\rangle$ and $|s_p,\lambda\rangle$ is given by[16]

$$Z_A = \int_0^\infty dt\; I_\lambda(t)\,, \qquad (6.33)$$

with

$$I_\lambda(t) \equiv \langle s_p|e^{-t(L_0+\widetilde{L}_0)}|D_{p+1}\rangle\,. \qquad (6.34)$$

The integrand of Z_A is a product $I_\lambda(t) = I^0_\lambda(t)I^S(t)$ of contributions from time components and spatial plus ghost components. The contribution from these latter components is the same as in the free theory, namely[17]

$$\begin{aligned} I^S(t) &= \frac{1}{2t}V_{p+1}\int\frac{d^{p+1}\vec{p}}{(2\pi)^{p+1}}e^{-\frac{2\pi\vec{p}^2}{t}-\frac{y^2}{2\pi t}}\sum e^{-2\pi(h_i-1)/t} \\ &= \frac{V_{p+1}}{2t}\left(\frac{8\pi^2}{t}\right)^{-(p+1)/2} e^{-\frac{y^2}{2\pi t}}\eta(i/t)^{-23} \\ &= \frac{V_{p+1}}{2}(8\pi^2)^{-(p+1)/2}t^{(p-24)/2}e^{-\frac{y^2}{2\pi t}}\eta(it)^{-23}\,. \end{aligned} \qquad (6.35)$$

The timelike components were computed in [26] (up to the factor of i from analytic continuation of the volume) as

$$I^0_\lambda(t) = \langle B|e^{-\pi t(L_0+\widetilde{L_0})}|N\rangle = \frac{iV_0}{\sqrt{2}}\sum_{j=0,1,\cdots} D^j_{00}(2\pi\lambda)\;\chi^{Vir}_j(e^{-2\pi t})\,, \qquad (6.36)$$

where

$$D^j_{00}(2\pi\lambda) = \frac{1}{j!}\frac{d^j}{d\xi^j}\left[\xi^j(1-\xi)^j\right], \qquad \xi = \sin^2(\pi\lambda)\,, \qquad (6.37)$$

and V_0 is the real volume in the X^0 direction and

$$\chi^{Vir}_j(q) = q^{-1/24}(q^{j^2}-q^{(j+1)^2})\prod_{n=1}^\infty\frac{1}{1-q^n} \qquad (6.38)$$

[16] To compute the force between the D-brane and the s-branes we must multiply this expression by a factor of 2, since the string can stretch in either orientation.

[17] In general when there is on-shell closed strings exchange one must include an ordering and $i\epsilon$ prescription for the t and $\vec{p}$ integrations. However when one of the boundary states is a D-brane, local energy conservation prohibits emission/absorption of an on-shell closed string.

is the Virasoro character. The total integrand from large t goes as

$$I_\lambda(t) = i\frac{V_0 V_{p+1}}{2}(8\pi^2)^{-(p+1)/2}\int dt\ t^{(p-24)/2}e^{-y^2/2\pi\alpha' t}(e^{2\pi t}+23+\cos(2\pi\lambda)+\cdots)\,. \quad (6.39)$$

When $\lambda = 0$ we have two D(p+1)-branes and (6.39) reduces to the usual expression. As usual the force between the D-brane and s-brane is obtained by differentiating with respect to y. As λ ranges from 0 to 1/2 and the D-brane goes to an s-brane and then an sD-brane, the force decreases by a factor of 11/12.

6.5 The finite-temperature annulus

In this subsection we compute the bosonic annulus diagram at general λ and temperature $T = 1/2\pi n$. We will find that the $T \to 0$ limit reproduces the results of the previous section.

At finite temperature $T = 1/2\pi n$ one naturally computes the euclidean thermal partition function. This is obtained by Wick rotation $X^0 \to i\phi$ with the euclidean time ϕ compactified on a circle of radius n [39, 37]. We consider a D(p+1)-brane located at $X^m = 0$, $m = p+2,\ldots,25$, with world-volume extending in the $X^1,\ldots,X^{p+1}$ directions and wrapped around the ϕ circle. There is also an sp-branes located at $X^m = y^m$ and parallel to the D-brane in the spatial directions. When λ vanishes, we simply have the euclidean annulus connecting two finite temperature D(p+1)-branes. The open string 1-loop calculation gives (before integrating over t)

$$\begin{aligned}\mathrm{Tr}\, e^{-2\pi L_0/t} &= \frac{1}{\eta(i/t)}\sum_m e^{-\frac{2\pi}{t}\frac{m^2}{n^2}} \\ &= \frac{1}{\eta(i/t)}\vartheta\left(0,\frac{2i}{n^2 t}\right) \\ &= \frac{n}{\sqrt{2}\eta(it)}\vartheta\left(0, in^2\frac{t}{2}\right). \end{aligned} \quad (6.40)$$

We will recover this result below in the special case $\lambda = 0$.

We will now turn to the euclidean s-brane boundary state. At the self-dual temperature ($n = 1$), the time component of the boundary state describing an s-brane is [26]

$$|B\rangle_{\mathrm{SU(2)}} = e^{i2\pi\lambda J_1}|N\rangle_{\mathrm{SU(2)}}\,. \quad (6.41)$$

For other values of n the boundary state describing an array of n s-branes is, up to a normalization factor, simply the projection of $|B\rangle_{\mathrm{SU(2)}}$ onto allowed momentum and winding modes. For $\lambda = \pm 1/2$, this boundary state describes n Dirichlet branes on a circle. The boundary state $|B\rangle$ at temperature $T = 1/2\pi n$ is

$$\begin{aligned}|B\rangle_{R=2\pi n} &= 2^{-1/4}\sum_{j=0,1/2,\cdots} P_n e^{i\theta^a J^a}|j,m,-m\rangle\rangle \\ &= 2^{-1/4}\sum_j\sum_{m,w} D^j_{m-wn,-m}(2\pi\lambda)|j,m,m-wn\rangle\rangle\,. \end{aligned} \quad (6.42)$$

At $\lambda = \pm 1/2$ equation (6.42) simplifies to

$$2^{-1/4} \sum_{j=0,1/2,\cdots} e^{\pm i\pi j} |j, m, m\rangle\rangle. \tag{6.43}$$

The Neumann boundary state at $T = 1/2\pi n$ is

$$|N\rangle_{R=2\pi n} = 2^{-1/4} \sum_{j=0,1/2,\cdots} \sum_{w} \left| j, \frac{wn}{2}, -\frac{wn}{2} \right\rangle\!\!\right\rangle. \tag{6.44}$$

Here the second sum is over allowed values of w, i.e. wn is restricted to be even (odd) when j is integer (half integer). The time component of the annulus amplitude is

$$\langle B| e^{-\pi t(L_0 + \widetilde{L_0})} |N\rangle_{R=2\pi n} = \frac{n}{\sqrt{2}} \sum_{j=0,1/2,1,\cdots} \sum_{w} D^{j}_{\frac{wn}{2}, \frac{wn}{2}}(2\pi\lambda)\ \chi^{Vir}_j(e^{-2\pi t}). \tag{6.45}$$

The prefactor n comes from the volume of the ϕ zero mode.

The trivial case $\lambda = 0$ corresponds to a Neumann boundary condition. We should be able to recover from (6.45) the answer for ordinary D-branes (6.40). We can write (6.45) as

$$\langle N| e^{-\pi t(L_0 + \widetilde{L_0})} |N\rangle_{R=2\pi n} = \frac{n}{\sqrt{2}} \sum_{j=0,1/2,1,\cdots} C_{n,j}\, \chi^{Vir}_j(e^{-2\pi t}), \tag{6.46}$$

where

$$C_{n,j} = \sum_{wn/2 \in \{-j, -j+1, \cdots, j\}} 1. \tag{6.47}$$

Suppose n is even. Then the $wn/2$ are integers, so the contribution to (6.47) comes from integer values of j. Equation (6.46) can then be rewritten as

$$\frac{n}{\sqrt{2}\eta(it)} \sum_{j=0}^{\infty} (C_{n,j} - C_{n,j-1}) e^{-2\pi j^2 t}. \tag{6.48}$$

For n even,

$$C_{n,j} - C_{n,j-1} = \begin{cases} 2\,, & j = kn/2, k \geq 1\,; \\ 1\,, & j = 0\,; \\ 0\,, & \text{otherwise} \end{cases} \tag{6.49}$$

so we can evaluate the sum (6.46)

$$\frac{n}{\sqrt{2}\eta(it)} \sum_{k=-\infty}^{\infty} e^{-\pi n^2 k^2 t/2} = \frac{n}{\sqrt{2}\eta(it)} \vartheta\left(0, \frac{in^2 t}{2}\right), \tag{6.50}$$

which agrees with (6.40). A similar analysis yields the same expression for odd n.

At the special values $\lambda = \pm 1/2$ only the $w = 0$ sector will contribute to the amplitude (6.45), so j is therefore restricted to be an integer. The amplitude (6.45) then reduces to

$$\frac{n}{\sqrt{2}} \eta(it)^{-1} \sum_{j=-\infty}^{\infty} e^{-2\pi t j^2 + \pi i j} = \frac{n}{\sqrt{2}\eta(it)} \vartheta\left(\frac{1}{2}, 2it\right). \tag{6.51}$$

Combining this with (6.35), we find that the annulus partition function at generic λ and n is

$$\int_0^\infty dt\ \langle s_p|e^{-\pi t(L_0+\widetilde{L}_0)}|D_{p+1}\rangle = \int_0^\infty dt\frac{nV_{p+1}}{2\sqrt{2}}(8\pi^2)^{-\frac{p+1}{2}}t^{\frac{p-24}{2}}e^{-\frac{y^2}{2\pi t}}\eta(it)^{-24}\times \qquad (6.52)$$
$$\times \sum_{j=0,1/2,1,\cdots}\sum_w D^j_{\frac{wn}{2},\frac{wn}{2}}(2\pi\lambda)\ (e^{-2\pi tj^2}-e^{-2\pi t(j+1)^2})\,.$$

For $\lambda=\pm 1/2$ and all n, the contribution from large t goes like

$$nV_{p+1}\int dt\ t^{(p-24)/2}e^{-y^2/2\pi\alpha' t}(e^{2\pi t}+22+\cdots)\,. \qquad (6.53)$$

As usual, the first term in the sum corresponds to the amplitude of tachyon exchange. The graviton exchange amplitude, given by the second term in the sum, falls off like $|y|^{p-22}=|y|^{2-(25-(p+1))}$, as in the zero-temperature case. This agrees with (6.39) up to the factor of i, which is due to the fact that (6.53) comes from the euclidean rather than lorentzian one loop diagram.

6.6 S-branes and D-instantons

In this subsection we discuss the relation between s-branes and D-instantons.

Instantons fall into two general categories: those with and those without a time reversal symmetry. An example of the latter is the Yang-Mills instanton. It describes tunneling between topologically distinct vacua. Any attempt to continue it to real time yields an imaginary solution. There is a topological lorentzian configuration — the creation and decay of a sphaleron — which interpolates between the distinct vacua, but it is not obtained by analytic continuation of the euclidean instanton solution.

The situation is different when there is a time reversal symmetry, as in the euclidean bubble describing the decay of the false vacuum. The analytic continuation to real time yields a real solution describing a contracting/expanding bubble of the true vacuum inside the false vacuum. Indeed, the semi-classical decay process is quantum tunneling followed by the real time expanding bubble solution. In this case both the euclidean and lorentzian solutions are real and meaningful.

The sD-branes (and most of the s-branes) discussed herein are of this latter character. They are processes in which the tachyon rolls up one side of the barrier and back down the same side, and so are time symmetric. While we have been focusing on the lorentzian solutions, there are also euclidean solutions which represent tunneling through the tachyon barrier at $\mathcal{T}=0$. For example, we could consider a semi-classical process in which the tachyon is incident on the barrier from $\mathcal{T}=-\infty$, tunnels through it, and then proceeds to $\mathcal{T}=+\infty$. The tunneling phase of this evolution would be described by the euclidean continuation of the s-brane. It connects two time-symmetric solutions in which $\mathcal{T}$ bounces off the barrier. Alternately the periodically identified instanton can be interpreted as a finite-temperature tunneling in which the energy comes from the thermal bath. Such tunneling processes could become important near the Hagedorn transition.

There are also of course non-time symmetric solutions with large enough energy to classically pass over the barrier. These solutions, largely the focus of [1], will result in a change in the RR s-charge Q_s evaluated at $t = \pm\infty$. They are analogs of baryon-number-violating sphaleron creation/decay in the standard model. They do not have a real continuation to euclidean space (at least by the usual method). In the superstring, one has a cosh rather than a sinh in (5.15), while in the bosonic string one has a sinh rather than a cosh in (2.4).

7. Timelike holography

An interesting potential application of s-branes is to the problem of finding a string theory configuration with a timelike holographic dual. We close this paper with some speculation on this topic.

The sD-brane boundary state for the superstring at temperature $T = 1/2\sqrt{2}\pi n$ describes $2n$ euclidean D-branes spaced along a circle at intervals $\sqrt{2}\pi$. According to (6.14), these have alternating RR charge, and hence are really n D-branes and n anti-D-branes. One may also consider beginning with a boundary interaction on N coincident lorentzian D(p+1)-branes, in which case the individual euclidean (anti) Dp-branes become replaced by a collection of N coincident (anti) Dp-branes.

According to the discussion of section 4, this finite temperature configuration determines an effective long-distance euclidean field theory. This field theory evidently contains $2n$ supersymmetric U(N) gauge theories. Additionally, the $\sqrt{2}\pi$ spacing is precisely such that the would-be tachyonic open string connecting a D-brane and an anti-D-brane is massless. (A similar massless mode appears in the bosonic sD-brane.) This couples the U(N) theories with bifundamentals in a manner that breaks supersymmetry.

Assuming they do exist, what could the dual supergravity solutions be? In part these should be determined by the symmetries. A number of potentially dual solutions have appeared in the literature. Many of them have an R-symmetry corresponding to Lorentz transformations transverse to the brane. This symmetry is clearly spontaneously broken in all the s-branes discussed herein, and hence should not appear in the supergravity solution. Some solutions that do have the appropriate symmetries have appeared in [19, 15, 18]. Particularly intriguing in this connection is a class of solutions [18] that exhibit thermal particle production and are periodic in euclidean time.

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